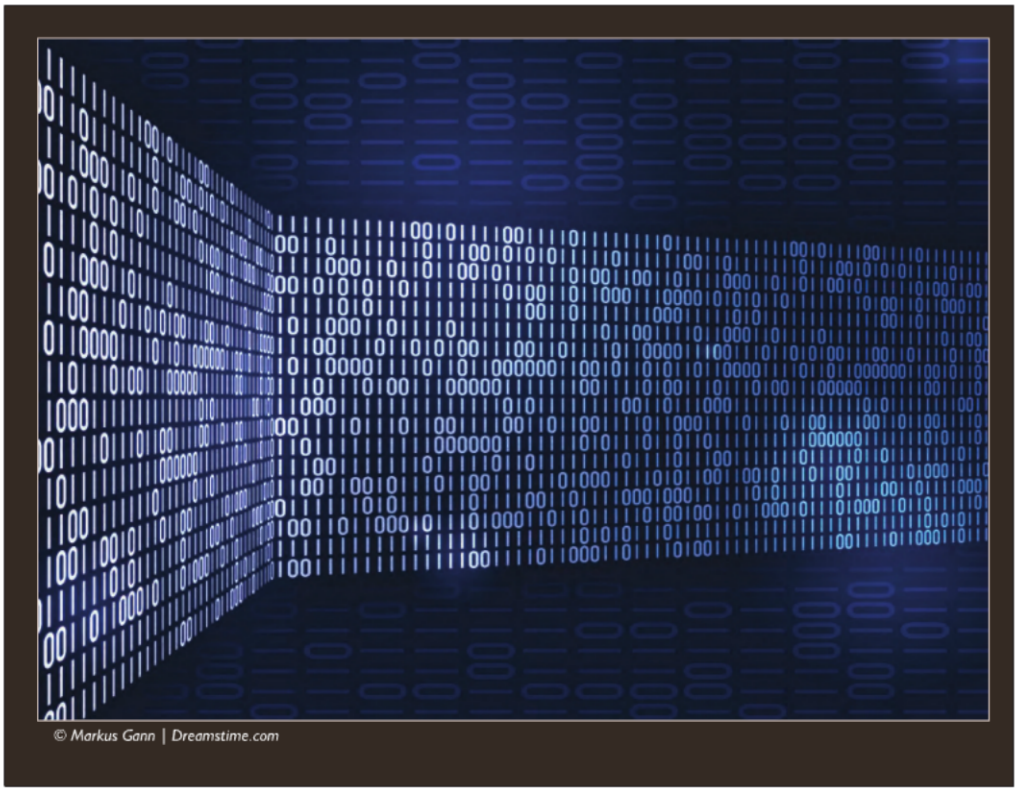
Twelfth in a series of LP surveys highlights new features, facilities that help address a broader variety of applications.



By Robert Fourer

Linear Programming

This is the twelfth in a series of *OR/MS Today* surveys of software for linear programming, dating back to 1990. As in the case of earlier surveys, information has been gathered by means of a questionnaire sent to software vendors by *OR/MS Today*. Results are summarized by product in the tables following this article. Further information is available directly from the vendors, for whom contact information is provided at the end.

Products listed in this survey are concerned, at the least, with minimizing or maximizing linear constraints subject to linear equalities and inequalities in numerical decision variables. All products provide for continuous variables that may take any values between their bounds, and many also accommodate integer variables that are limited to whole-number values in some way. The respectively continuous and discrete problems that use these variables are commonly distinguished as linear programs (LPs) and integer or mixed-integer linear programs (IPs/ILPs or MIPs/MILPs), but for convenience "LP software" is used herein as a general term for the packages covered, and "LP" refers to linear problems that may or may not have some integer variables.

Some of the listed products handle other kinds of discrete variables and constraints, as well as varied nonlinearities and even problems outside of optimization. This survey focuses on developments and trends in the linear programming and related integer programming aspects of the software, however. Also, the listing excludes products that address only certain applications or formulations of LP, or that are not targeted to large LP instances, as these products are more properly evaluated in the context of other broad categories of optimization software. The ordering of topics below is roughly parallel to the organization of the tables, and terms in bold correspond to table headings.

The printed table is limited to responses available by press time, but additional responses are welcome and will be added to the online version of the survey listing. To learn more, write to Online Projects Manager Patton McGinley, patton@lionhrtpub.com, or go directly to the survey at www.lionhrtpub.com/ancill/lpsurvey2013.shtml.

Types of Packages

Although the products surveyed have a common purpose and share many aspects of design, they are best understood as incorporating two complementary but fundamentally different types of software. **Solver** software takes an instance of an LP model as input, applies one or more solution methods and returns the results. **Modeling** software mediates between human modelers and solvers, providing general and intuitive ways to express symbolic models, and offering features for importing data, generating problem instances, invoking solvers, analyzing results, scripting extended algorithmic schemes and interfacing to broader applications. Products of this latter kind are typically built around a computer modeling language either designed specifically for describing optimization models or adapted from the features of an already popular programming or scripting language.

Numerous solver and modeling products have been developed as **independent applications.** Thus, solvers typically support **links to** many modeling systems, and modeling systems offer **links to** many solvers. In some cases the two may be acquired as separate products and linked by the purchaser, but more commonly they are bought in bundles of various kinds. Several developers of general-purpose modeling systems arrange to offer a variety of bundled solvers, providing modelers with an easy way to benchmark competing solvers before committing to purchase one. Some solver developers also offer bundles with modeling systems. A number of the latter developers also offer **integrated** systems that provide a modeling environment specifically for their own solvers. Many variations on these arrangements are possible, so prospective purchasers are advised to confirm the details carefully.

Interfaces to Other Software

Since optimization models are usually developed in the context of some larger algorithmic scheme or application (or both), the ability of LP software to be embedded is often a key consideration. Thus, although virtually any of the listed products can be run as an **independent application** in some kind of standalone mode, many are available as callable programs, generally in the form of **class libraries** in an object-oriented framework. About half a dozen general programming languages of varying designs are widely supported for this purpose; some products also offer their own specialized application development tools.

Solver systems have long been available as program libraries, allowing them to be embedded within application-specific systems and interfaces. As solver library designs have evolved, some have taken on aspects of symbolic modeling, such as algebraic specification of constraints. At the same time, modeling systems have been extended to offer their own program libraries, so that the considerable advantages of developing and maintaining a modeling language formulation can be carried over into application software that solves instances of a model. It is possible to embed an entire modeling system, or a particular model or an instance of a model; not all systems provide all possibilities, so some study is necessary to determine which products are right for a given project in this respect.

Most commercial LP software libraries are distributed as binaries for linking into the user's applications. In addition, our table includes numerous solvers that make their **source code** freely available under one of the standard open source licenses; many of these are available through the COIN-OR repository (www.coin-or.org). Open source is ideal in situations where budgets are tight or where the greatest degree of flexibility is required, such as in creating new algorithms and algorithmic schemes. Many of the open-source solvers are also available as precompiled binaries for the more popular platforms.

The application development environments provided by **spreadsheet** and **database** programs have proved to be particularly useful tools for embedding LP software. At the least, most LP modeling environments can read and write common spreadsheet and database file formats. Spreadsheet packages can also accept solver add-ins whose appeal to users and convenience for development are widely appreciated. The solver add-in that comes packaged with Excel is effective for relatively small and easy problems; independent developers offer much more powerful spreadsheet optimizers. Some can work with a variety of spreadsheet functions that go beyond the smooth arithmetic functions assumed by classical optimization software. Several scientific and statistical packages also offer LP software add-ins specifically for their products.

Virtually all LP modeling systems and solvers can also handle model instances expressed in simple text formats, especially the "MPS" format dating back many decades and various "LP" formats that resemble textbook examples complete with + and = signs. These formats mainly serve for submitting bug reports and for communicating benchmark problems. Modeling systems use much more general and efficient formats for communicating problem instances to solvers and for retrieving results, but each has adopted its own format, and efforts at standardization are still at an early stage.

Platforms

Among operating system platforms, **Windows** remains ubiquitous, while **Linux** and **MacOS** are very widely supported as well. Several other Unix variants appear frequently among **Other OSs** (and indeed many LP systems support MacOS through its underlying Unix-based shell). Several products offer tools to help modelers create Web-based applications; app development tools for tablets or phones seem a likely next step, but none seem to be in release as yet.

Solvers offering **multiprocessor** versions for **shared memory** have become common, as the number of cores available in off-the-shelf PCs continues to increase. Indeed the automatic use of all available cores has become routine for some purposes, such as the exploration of search trees in MIP solvers. Support for **distributed memory** remains relatively less common, perhaps due to the greater difficulty in achieving a favorable cost-performance tradeoff while supporting a diversity of high-end computing architectures.

Software Survey, *continued on p.52*

	Software Description															Platforms Supported						Pricing Information							
	Type					Form															Multicore & Multiprocessor Support (List Platforms)		Commercial			Educational			Student/Academic	
						Procedure Class Library																								
	Solver	Modeling Environment	Integrated Solver & Modeling Environ.	Independent Application	C	C++	.NET	Java	Python	MATLAB	R	Other	Source Code	Language?	Add-in To:	PC/Windows	PC/Linux	Mac OS X	Other (specify)	Shared Memory	Distributed Memory	Single Machine	Floating Licenses available	Site Licenses available	Single Machine	Floating Licenses available	Site Licenses available	Unlimited	Size Restricted	
AIMMS *Paragon Decision Technology, Inc.*	y	y	y	y	y	y	y	y	–	–	–	–	–	–	MS Excel, Open Office Calc	y	y	–	–	Cplex, Gurobi, Mosek	–	Contact	y	y	Free	y	y	Free	Free	
AMPL *AMPL Optimization Inc*	–	y	–	y	–	–	–	–	–	–	–	–	–	–	–	y	y	y	HP-UX, Linux/ia 64, Linux/P PC, Solaris	–	–	$4,000	y	–	$400	y	–	For courses	Yes	
Analytic Solver Platform *Frontline Systems Inc.*	–	–	y	–	–	–	–	–	–	–	–	VBA in Excel	–	–	Microsoft Excel	y	–	–	–	Windows	Windows HPC Server	$5,495	y	–	$1,375	y	–	–	$60 to $90, free w/ certain text-books	
Analytica *Lumina Decision Systems*	y	y	y	y	–	–	–	–	–	–	–	–	–	–	–	y	–	–	–	–	–	$4,995	y	y	$2,645	y	y	–	–	
CBC (COIN-OR Branch and Cut) *COIN-OR Foundation*	y	–	–	y	–	y	–	–	y	y	–	–	y	C++	MS Excel (Open-Solver)	y	y	y	–	All	–	$0	–	–	$0	–	–	$0	–	
CHiPPS *COIN-OR*	y	–	–	–	–	y	–	–	–	–	–	–	–	–	–	y	y	y	Sun Solarix	MPI message-passing system	MPI message-passing system	Free	–	–	Free	–	–	Free	No	
CLP *COIN-OR Foundation*	y	–	–	y	–	y	–	–	y	y	–	–	y	C++	–	y	y	y	–	All	–	$0	–	–	$0	–	–	$0	–	
CMPL **(<Coliop | Coin> Mathematical Programming Language)** *Technical University of Applied Sciences Wildau*	–	y	y	y	–	–	–	y	y	–	–	–	y	–	–	y	y	y	–	–	–	Open Source - GPLv3	–	–	Open Source - GPLv3	–	–	Open Source - GPLv3	–	
CoinMP Open-Source Solver *Maximal Software, Inc.*	y	–	y	–	y	y	y	–	–	–	–	–	y	C and C++	MPL Modeling System, Apache OpenOf-fice, et al	y	y	y	Solaris	–	–	Free through the Open-Source CLP Lic.	–	–	Free through the Open-Source CLP Lic.	–	–	Free through the Open-Source CLP Lic.	–	
Coopr *Open Source* *(sponsored by Sandia National Laboratories and COIN-OR)*	y	y	y	y	–	–	–	–	y	–	–	–	y	Python	–	y	y	y	Compati-ble with Python 2.6 and 2.7	–	–	N/A (open source)	–	–	N/A	–	–	N/A	N/A	
Couenne *COIN-OR*	y	–	–	y	–	y	–	–	–	–	–	–	y	C++	–	y	y	y	–	–	–	Open Source, Eclipse Public License	–	–	Open Source, Eclipse Public License	–	–	Open Source, Eclipse Public License	–	

Data Compatibility					Solvers or Modeling Environments			Formulations Supported											Algorithms/Methods							Cloud /Remote Access Availability			New Features (Since June 2011)	Other Techniques (Available as part of the package)
								Variable Types					Constraint & Objective Types																	
Reads Spreadsheet Files	Writes Spreadsheet Files	Reads Database Files	Writes Database Files	Reads and Writes Text Files	Solvers or Modeling Environments that Link to this Product	Bundeled as a Single Package	Available Separately	Integer, Binary	Semi-continuous, Semi-integer	Arbitrary Discrete (SOS1)	Piecewise Linear (SOS2)	Other	Convex Quadratic	Conic Quadratic	General Convex	General Nonlinear	Other	Primal Simplex	Dual Simplex	Interior-Point	Branch-and-Cut	Presolve	Heuristic Search	Infeasibility Analysis	Other	NEOS	Amazon	Other		
y	y	y	y	y	Cplex, Gurobi, Mosek, Xa, Cbc, Conopt, Knitro, Snopt, Minos, Ipopt, Baron, Lgo, et al	y	y	y	y	y	y	Stochastic, Adjustable (robust), Element Variable (CP)	y	y	y	y	Indicator, Lazy, Cut, Complementarity, Global (CP), Scheduling	y	y	y	y	y	–	y	Robust Counterpart, Stochastic, Parallel Solving, etc.	–	–	–	Constraint Programming, Uncertainty Constraints, Automatic Benders decomposition, a.o.	Various GUI improvements (such as New IDE, extended Pivot table features), Data change management functions, a.o.
y	y	y	y	y	CPLEX, Gurobi, Xpress, CONOPT, KNITRO, MINOS, others listed at www.ampl.com/solvers.html	y	y	y	y	y	y	–	y	y	y	y	Complementarity, logical	y	y	y	y	y	y	y	Constraint (CP) solvers	y	–	–	Extended function library, Logic and constraint programming (CP) extensions, Expanded database support	General nonlinear support incl automatic 1st & 2nd derivatives; solver directives; scripting language; user-defined functions
y	y	y	y	y	5 built-in, 8 plug-in Solvers including Gurobi, Xpress-MP, KNITRO, Mosek, SNOPT	y	y	y	y	y	–	Alldifferent constraints	y	y	y	y	General non-smooth non-convex; chance constraints	y	y	y	y	y	y	y	Robust optimization, stochastic decomposition, sim. optimization	–	–	Windows Azure using Solver Server	New in 2013, integrated predictive and prescriptive analytics + visualization, superset of Risk Solver Platform and XLMiner	Nonlinear, global, non-smooth optimization; simulation/risk analysis, decision trees; data mining, classification, prediction
y	y	y	y	y	Uses Frontline premium solver engines	y	–	y	y	y	y	–	y	y	y	y	–	y	y	y	y	y	y	y	–	–	–	–	Structured Optimization - flexible, transparent and multidimensional paradigm; Expression Assist, Analytica Cloud Player	Integrated with decision analytic and uncertainty analysis techniques. Intelligent arrays. Influence diagrams.
–	–	–	–	y	AMPL, GAMS, MPL, AIMMS, PuLP, Pyomo	–	y	y	y	y	y	–	–	–	–	–	–	y	y	–	y	y	y	–	–	y	–	–	–	–
–	–	–	–	y	–	y	–	y	–	–	–	–	–	–	–	–	–	y	y	–	y	y	y	–	–	–	–	–	–	–	–
–	–	–	–	y	AMPL, GAMS, MPL, AIMMS, PuLP, Pyomo	–	y	–	–	–	–	–	y	–	–	–	–	y	y	y	–	–	–	–	–	–	y	–	–	–	–
–	–	–	–	y	–	y	y	y	y	y	y	Product of variables with at least one integer factor	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CMPLServer (available from summer 2013)	API's for Python (pyCMPL) and Java (jCMPL) / CMPLServer for distributed optimizations (available from summer 2013)	CMPL executes the COIN-OR OSSolverService, GLPK, Gurobi, SCIP and CPLEX directly to solve the generated model instance.
–	–	–	–	–	MPL Modeling System, CLP, CBC	y	y	y	y	y	y	–	–	–	–	–	–	y	y	y	y	y	y	y	–	–	–	–	New release CoinMP 1.6 offers library interfaces for C#, VB.Net. Linux/UNIX versions with automake/configure support.	CoinMP is available as precompiled CoinMP.dll and CoinMP.so (Linux/Unix) libraries that can be easily embedded into projects.
y	–	y	–	y	CPLEX, Gurobi, CBC, IPOPT, GLPK, PICO, XPRESS, SCIP, OpenOpt solvers, asl-compliant solvers, ph	–	y	y	–	y	y	Disjunctive programming	y	y	y	y	–	–	–	–	–	–	–	–	ph mixed-integer stochastic programming heuristic	–	–	–	OpenOpt solvers, runtime and memory efficiency improvements, support for modeling blocks, set expressions, ODBC support	New support for modeling with differential algebraic equations has been prototyped in the latest release.
–	–	–	–	–	AMPL, GAMS	y	y	y	y	–	–	–	y	y	y	y	–	y	y	–	y	y	y	–	–	y	–	–	Feasibility pump heuristic, SDP relaxation-based cuts	–

	Software Description															Platforms Supported					Pricing Information							
	Type					Form														Multicore & Multiprocessor Support (List Platforms)		Commercial			Educational			Student/Academic	
						Procedure Class Library																							
	Solver	Modeling Environment	Integrated Solver & Modeling Environ.	Independent Application	C	C++	.NET	Java	Python	MATLAB	R	Other	Source Code	Language?	Add-in To:	PC/Windows	PC/Linux	Mac OS X	Other (specify)	Shared Memory	Distributed Memory	Single Machine	Floating Licenses available	Site Licenses available	Single Machine	Floating Licenses available	Site Licenses available	Unlimited	Size Restricted	
CPLEX Optimization Studio 12.5 *IBM Corporation*	–	–	y	y	y	y	y	y	y	y	–	–	–	–	–	y	y	y	AIX, HP-UX, Solaris; see website	see website	see website	Contact	y	y	No cost; see Website	y	y	see Website	–	
DIP (Decomposition for Integer Programming) *COIN-OR Foundation*	y	y	y	–	–	y	–	–	y	–	–	–	y	C++, Python	–	y	y	y	–	All	–	$0	–	–	$0	–	–	$0	–	
Frontier Analyst *Banxia Software Ltd.*	–	–	y	y	–	–	–	–	–	–	–	SOAP interface	–	–	–	y	–	–	–	Windows multi-core	–	From 395 GBP	y	y	From 195 GBP	y	y	–	–	
GAMS *GAMS Development Corporation*	–	y	y	–	y	y	y	y	y	–	–	–	–	–	–	y	y	y	AIX, SUN Solaris,	All	All	see Website	y	–	see Website	y	–	–	see Website	
GCG *Zuse Institute Berlin & Operations Research, RWTH Aachen University*	y	–	–	y	y	y	–	–	–	–	–	–	y	C, C++	–	–	y	–	–	–	–	Free under the LGPL, but SCIP required	–	–	Free	–	–	–	–	
GENO *Apex Research Limited*	y	–	–	–	–	–	–	–	–	–	–	–	–	–	GAUSS, TOMLAB, GAMS	y	y	y	–	–	–	Contact	–	–	Contact	–	–	Contact:	–	
GIPALS - Linear Programming Environment *Optimalon Software Ltd.*	y	y	y	y	–	–	–	–	–	–	–	–	–	–	–	y	–	–	–	–	–	$197	y	y	$97	y	y	–	$1,000 variables and constraints	
Gipals32 - Linear Programming Library *Optimalon Software Ltd.*	y	–	–	–	y	y	y	–	–	–	–	Single Windows DLL	–	–	–	y	–	–	–	–	–	$179	y	y	$80	y	y	–	$1,000 variables and constraints	
Global Optimizer *Vanguard Software*	y	y	y	y	–	–	–	–	–	–	–	Web Services (SOAP/WSDL)	–	–	–	y	–	–	–	Windows	Windows	Call for pricing	y	y	50% discount	y	y	–	–	
GLPK (GNU Linear Programming Kit) *Free Software Foundation, Inc.*	y	y	y	y	y	–	–	–	–	–	–	–	y	ANSI C 89	–	y	y	y	MMIX, a 64-bit RISC by Donald Knuth	–	–	–	–	–	–	–	–	–	–	
GNU Linear Programming Kit (GLPK) *Free Software Foundation, Inc.*	y	y	y	y	y	–	–	–	–	–	–	–	y	ANSI C 89	–	y	y	y	MMIX 64-bit RISC by Donald Knuth	–	–	–	–	–	–	–	–	–	–	

Data Compatibility					Solvers or Modeling Environments			Formulations Supported									Algorithms/Methods								Cloud /Remote Access Availability			New Features (Since June 2011)	Other Techniques (Available as part of the package)
								Variable Types					Constraint & Objective Types																	
Reads Spreadsheet Files	Writes Spreadsheet Files	Reads Database Files	Writes Database Files	Reads and Writes Text Files	Solvers or Modeling Environments that Link to this Product	Bundeled as a Single Package	Available Separately	Integer, Binary	Semi-continuous, Semi-integer	Arbitrary Discrete (SOS1)	Piecewise Linear (SOS2)	Other	Convex Quadratic	Conic Quadratic	General Convex	General Nonlinear	Other	Primal Simplex	Dual Simplex	Interior-Point	Branch-and-Cut	Presolve	Heuristic Search	Infeasibility Analysis	Other	NEOS	Amazon	Other		
y	y	y	y	y	AMPL, GAMS, AIMMS	–	y	y	y	y	y	–	y	y	–	–	–	y	y	y	y	y	y	y	–	–	–	–	Performance improvements; QCP duals; CPLEX remote object for distributed parallel algorithms; others too numerous to name	–
–	–	–	–	y	PuLP/DipPy	y	–	y	–	–	–	–	–	–	–	–	–	y	y	–	y	–	–	–	–	–	–	–	–	Automatic block structure detection, decomposition, column generation, lagrangian relaxation
y	y	y	–	y	–	–	–	–	–	–	–	–	–	–	–	–	–	y	–	–	–	–	–	–	–	–	–	PowerTrain engine designed for internal corporate deployment	PowerTrain - a web service to provide analytical capabilities to your dashboard or web site application.	Implements all common Data Envelopment Analysis (DEA) models.
y	y	y	y	y	–	y	y	y	y	y	y	–	y	y	y	y	–	y	y	y	y	y	y	y	–	y	y	–	http://www.gams.com/docs/release/release.htm	–
–	–	–	–	y	–	–	–	y	–	–	–	–	–	–	–	–	–	–	–	–	–	y	y	–	Branch-and-Price, automatic Dantzig-Wolfe reformulation	–	–	–	First release in August 2012; Branch-Cut-and-Price, heuristics and automatic detection of classical DW structure.	–
–	–	y	y	y	GAUSS, TOMLAB, (GAMS, forthcoming)	–	y	y	y	y	y	–	y	y	y	y	Procedural objectives	–	–	–	–	–	–	–	Evolutionary Algorithm	–	–	–	Improved operators; solves nonlinear equations; automated parameter specification; provision to specify search points	–
–	y	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	y	–	y	y	–	–	–	–	–	–	Import and Export MPS Files.
–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	y	–	y	–	–	–	–	–	–	–	–
y	y	y	y	y	Vanguard System. Any others via Web Services (SOAP/WSDL)	–	y	y	y	y	y	Custom	y	y	y	y	Custom	y	–	–	y	y	y	y	Custom	–	–	SaaS (hosted by Vanguard)	–	Nonlinear optimization, discontinuous optimization, stochastic/simulation optimization
y	y	y	y	y	–	–	–	y	–	–	–	–	–	–	–	–	–	y	y	y	y	y	y	–	SAT preprocessor and solver (based on MiniSat)	y	–	–	–	–
y	y	y	y	y	–	–	–	y	–	–	–	–	–	–	–	–	–	y	y	y	y	y	y	–	SAT preprocessor and solver (based on MiniSat)	–	–	–	–	–

	Software Description															Platforms Supported						Pricing Information							
	Type				Form																Multicore & Multiprocessor Support (List Platforms)		Commercial			Educational			Student/Academic	
	Solver	Modeling Environment	Integrated Solver & Modeling Environ.	Independent Application	Procedure Class Library								Source Code	Language?	Add-in To:	PC/Windows	PC/Linux	Mac OS X	Other (specify)	Shared Memory	Distributed Memory	Single Machine	Floating Licenses available	Site Licenses available	Single Machine	Floating Licenses available	Site Licenses available	Unlimited	Size Restricted	
					C	C++	.NET	Java	Python	MATLAB	R	Other																		
Gurobi Optimization *Gurobi Optimization, Inc.*	y	y	y	y	y	y	y	y	y	y	y	Excel	–	–	–	y	y	y	AIX	All supported platforms	–	See website	y	y	Free	y	y	Yes	–	
IBM ILOG CPLEX Optimization Studio *IBM*	y	–	y	–	y	y	y	y	y	y	–	–	y	C	–	y	y	y	HP, AIX, z/OS, zLInux, pLinux	Windows, Linux, Mac OS, HP, AIX, z/OS, zLInux, pLinux	–	Contact	–	–	it's free for Academic/Educational use	–	–	Yes, no functional limitation	No size restriction	
Ipopt *COIN-OR*	y	–	–	–	y	y	y	y	y	y	y	Python, Optimization Services	y	C++	–	y	y	y	–	–	–	$0	–	–	$0	–	–	$0	$0	
KNITRO *Ziena Optimization LLC*	y	–	–	–	y	y	–	y	y	y	–	–	–	–	–	y	y	y	–	Windows, Linux, MacOSX	–	Contact	y	y	Contact	y	y	–	–	
LINDO API *LINDO Systems, Inc.*	y	–	–	y	y	y	y	y	–	y	–	VB, VBA, FORTRAN, Delphi	–	–	What's-Best!, LINGO, GAMS	y	y	y	Solaris	All supported platforms	–	Varies / options from $395	–	y	Free educational research licenses available	–	y	Yes	Yes	
LINGO *LINDO Systems, Inc.*	–	–	y	y	y	y	y	y	–	y	–	VB, VBA, FORTRAN, Delphi	–	–	Excel	y	y	–	–	All supported platforms	–	Varies / options from $495	–	y	Free educational research licenses available	–	y	Yes	Yes	
LOQO *Princeton University*	y	–	–	y	y	y	–	–	–	–	–	–	–	–	AMPL, GAMS	y	y	y	–	–	–	$2,000	y	y	$300	y	y	–	$200 x $200	
LPL Modeling System *Virtual Optima*	–	y	y	y	y	y	–	y	–	–	–	–	–	–	–	y	y	–	–	–	–	Contact	–	–	Contact	–	–	–	Contact	
MOSEK *MOSEK ApS*	y	–	–	y	y	y	y	y	y	y	y	–	–	–	–	y	y	y	–	All supported platforms	–	See Website	y	y	See Website	y	y	Free. See Website	–	
MPL Modeling System *Maximal Software, Inc.*	–	y	y	y	y	y	y	y	y	y	–	–	–	–	Cplex, Gurobi, Xpress, XA, Sulum, Lindo, et al	y	y	y	–	Windows, Linux, Unix	–	Free for development purposes, apply online	y	y	Free for academic use, apply online	y	y	Free by joining MPL Academic Prog.	Student version: $300x $300	
OMP Plus *OM Partners*	y	y	y	y	–	–	–	–	–	–	–	–	–	–	–	y	–	–	–	PC/ Windows	–	Contact	y	y	Contact	y	y	Contact	Contact	

	Data Compatibility					Solvers or Modeling Environments			Formulations Supported											Algorithms/Methods								Cloud /Remote Access Availability			New Features (Since June 2011)	Other Techniques (Available as part of the package)
									Variable Types					Constraint & Objective Types																			
	Reads Spreadsheet Files	Writes Spreadsheet Files	Reads Database Files	Writes Database Files	Reads and Writes Text Files	Solvers or Modeling Environments that Link to this Product	Bundeled as a Single Package	Available Separately	Integer, Binary	Semi-continuous, Semi-integer	Arbitrary Discrete (SOS1)	Piecewise Linear (SOS2)	Other	Convex Quadratic	Conic Quadratic	General Convex	General Nonlinear	Other	Primal Simplex	Dual Simplex	Interior-Point	Branch-and-Cut	Presolve	Heuristic Search	Infeasibility Analysis	Other	NEOS	Amazon	Other		
	y	y	y	y	y	AIMMS, AMPL, Frontline Systems, GAMS, MPL, CVX, etc.	y	y	y	y	y	y	–	y	y	–	–	Lazy constraints	y	y	y	y	y	y	y	Sifting, feasrelax, sensitivity analysis	y	y	Compute server	Compute server, tuning tool, interior point for QCP, MIQCP support, and Python modeling interface	–
	y	y	y	y	y	OPL language and integrated development Environment (aka Studio), with different optimization services	y	–	y	y	y	y	Quadratic constraints with quadratic goals	y	y	y	–	NLP can be solved with the constraint programming engine	y	y	y	y	y	y	y	Tuning, automatic conflict resolution	–	–	Optimization Studio	OptimServer + Server can be used from IDE SPSSModeler connector MIQCP solver, KPI testharness, remote obj.for distr.||alg.	Constr.prog., constraint-based scheduling (both in ILOG CP Optimizer=CPO), lexocographically ordered multi-criteria obj.
	–	–	–	–	–	AIMMS, AMPL, GAMS, OPTI, SciLab	y	y	–	–	–	–	–	y	–	y	y	–	–	–	y	–	–	–	–	–	y	–	–	New interface to linear solver HSL_MA97; improvements in interfaces to other HSL routines and build of Matlab interface	slpopt: toolbox that uses NLP sensitivity theory to generate fast approximations to solutions when problem parameters change
	–	–	–	–	–	AMPL, AIMMS, GAMS, Frontline Systems, MPL, MATLAB, Mathematica	y	y	y	–	–	–	–	–	y	y	y	Complementarity constraints	–	y	y	y	y	–	–	–	y	–	–	Various parallel features for multi-core platforms. Presolver. Python interface.	–
	–	–	–	–	y	What'sBest!, LINGO, GAMS	y	y	y	y	y	y	–	y	y	y	y	Can explicitly identify convex constraints, Geometric Programs	y	y	y	y	y	y	y	Stochastic & Chance Constraints, Guaranteed Global, Nonlinear Least Squares	–	–	–	Support for 200+ mathematical and probability functions, histogram generation, branch-and-price solver, multicore support.	Genetic algorithm, decomposition algorithm, Sprint/sift technique, auto detect problem type.
	–	–	–	–	y	–	–	–	y	y	y	y	–	y	y	y	y	Can explicitly identify convex constraints, Geometric Programs	y	y	y	y	y	y	y	Stochastic & Chance Constraints, Guaranteed Global, Nonlinear Least Squares	–	–	–	Support for 200+ mathematical and probability functions, histogram generation, branch-and-price solver, multicore support.	Genetic algorithm, decomposition algorithm, Sprint/sift technique auto detect problem type.
	–	–	–	–	–	AMPL	–	y	–	–	–	–	–	y	y	y	y	–	–	–	y	–	–	–	–	–	y	–	–	–	–
	y	y	y	y	y	–	–	y	y	–	y	y	–	y	y	y	y	–	–	–	–	–	–	y	–	–	–	–	–	Generate SVG-files	FastReport generator
	–	–	–	–	y	AIMMS, GAMS-Frontline Solver, CVX, Yalmip Rome	y	y	y	–	–	–	–	y	y	y	–	Semi-definite optimization	y	y	y	y	–	–	y	Primal network simplex	y	–	–	Support for semi-definite optimization and a new model orientated API called Fusion.	–
	y	y	y	y	y	Cplex, Gurobi, Xpress, XA, Sulum, Lindo, Mosek, CoinMP, Glpk, LPSolve, Conopt, Knitro, et al	y	y	y	y	y	y	Stochastic	y	y	y	y	Global	y	y	y	y	y	y	y	Stochastic	–	y	Our MPL Server version can be used to setup your own optimization cloud server.	New release MPL 4.2n, offers a large MPL C-Api Callable Library with over 500 functions, and full support for Multi-threading	With the new Optimax 2.0 Library, MPL models can be embedded into end-user applications using C#, C/C++, VB, Python, and Java
	y	y	y	y	y	–	–	–	y	y	–	–	–	y	–	–	–	–	y	y	y	y	y	y	y	–	–	–	–	Strong improvements in presolve and in branch-and-cut	Leading integrated solution for all planning related issues, from the strategic down to the operational level.

	Software Description															Platforms Supported						Pricing Information								
	Type				Form															Multicore & Multiprocessor Support (List Platforms)		Commercial			Educational			Student/Academic	
	Solver	Modeling Environment	Integrated Solver & Modeling Environ.	Independent Application	Procedure Class Library: C	C++	.NET	Java	Python	MATLAB	R	Other	Source Code	Language?	Add-in To:	PC/Windows	PC/Linux	Mac OS X	Other (specify)	Shared Memory	Distributed Memory	Single Machine	Floating Licenses available	Site Licenses available	Single Machine	Floating Licenses available	Site Licenses available	Unlimited	Size Restricted	
OptiMax Component Library *Maximal Software, Inc.*	–	y	y	–	y	y	y	y	y	y	–	–	–	–	Cplex, Gurobi, Xpress, XA, Sulum, Lindo, et al	y	y	y	–	Windows, Linux, Unix	–	Free for development purposes, apply online	y	y	Free for academic use, apply online	y	y	Free by joining MPL Academic Prog.	Student version: $300x $300	
Oracle Crystal Ball Suite *Oracle America Inc.*	y	y	y	–	–	–	y	–	–	–	–	–	–	–	MS Excel	y	–	–	–	–	–	Contact Vendor	–	y	Contact Vendor	–	y	–	–	
Premium Solver Platform *Frontline Systems Inc.*	–	–	y	–	–	–	–	–	–	–	–	VBA in Excel	–	–	Microsoft Excel	y	–	y	–	Windows	Windows HPC Server	–	y	–	$495	y	–	–	–	
pulp *Open Source*	–	y	–	–	–	–	–	–	y	–	–	–	y	python	–	y	y	y	–	–	–	Free	–	–	Free	–	–	Free	Free	
QMS - Quantitative Methods Software *QuantMethods*	–	–	y	y	–	–	–	–	–	–	–	–	–	–	–	y	y	y	–	–	–	Quote	–	y	$19.95 for 6 month subscription	–	y	–	–	
Risk Solver Platform *Frontline Systems Inc.*	–	–	y	–	–	–	–	–	–	–	–	VBA in Excel	–	–	Microsoft Excel	y	–	–	–	Windows	Windows HPC Server	$3,995	y	–	$1,000	y	–	–	$50 to $75, free w/ certain textbooks	
SAS *SAS Institute Inc.*	y	y	y	y	–	–	–	–	–	–	–	–	–	SAS	–	y	y	–	AIX, HP-UX, Solaris, z/OS	Windows, Linux, AIX, HP-UX, Solaris, z/OS	Windows, Linux, AIX, HP-UX, Solaris, z/OS	Contact	y	y	Contact	y	y	Contact	Not size-restricted	
SCIP *Zuse Institute Berlin*	y	–	y	y	y	y	–	–	y	y	–	–	y	C	–	y	y	y	–	Within UG framework (beta), all platforms	Within UG framework (beta), all platforms	Contact	y	y	Free	y	y	Free	–	
Solver SDK Platform *Frontline Systems Inc.*	–	–	y	–	y	y	y	y	–	y	y	VB/COM, PHP, JavaScript	–	–	–	y	y	–	–	Windows, Linux	Windows HPC Server	$995 to $1,995	y	–	$225 to $450	y	–	–	–	
SolverStudio	–	–	y	–	–	–	–	–	–	–	–	–	–	–	Excel	y	–	–	–	–	–	Free	–	–	Free	–	–	Free	–	
SoPlex *Zuse Institute Berlin*	y	–	–	y	–	y	–	–	–	–	–	–	y	C++	–	y	y	y	–	–	–	Contact	y	y	Free	y	y	No restrictions	No restrictions	

	Data Compatibility					Solvers or Modeling Environments			Formulations Supported											Algorithms/Methods								Cloud /Remote Access Availability			New Features (Since June 2011)	Other Techniques (Available as part of the package)
									Variable Types					Constraint & Objective Types																			
	Reads Spreadsheet Files	Writes Spreadsheet Files	Reads Database Files	Writes Database Files	Reads and Writes Text Files	Solvers or Modeling Environments that Link to this Product	Bundeled as a Single Package	Available Separately	Integer, Binary	Semi-continuous, Semi-integer	Arbitrary Discrete (SOS1)	Piecewise Linear (SOS2)	Other	Convex Quadratic	Conic Quadratic	General Convex	General Nonlinear	Other	Primal Simplex	Dual Simplex	Interior-Point	Branch-and-Cut	Presolve	Heuristic Search	Infeasibility Analysis	Other	NEOS	Amazon	Other		
	y	y	y	y	y	Cplex, Gurobi, Xpress, XA, Sulum, Lindo, Mosek, CoinMP, Glpk, LPSolve, Conopt, Knitro, et al	y	y	y	y	y	y	Stochastic	y	y	y	y	Global	y	y	y	y	y	y	y	Stochastic	–	y	–	New release Optimax 2.0 for CSharp, VB.Net, Python and Matlab with extensive new objects, and full multi-threaded support.	OptiMax 2.0 is an object-oriented Component Library, designed for embedding optimization models into end-user applications.
	y	y	y	y	y	–	–	–	y	y	–	–	–	y	y	y	y	–	–	–	–	–	–	y	y	Tabu Search	–	–	–	Improvement in the optimization algorithm.	–
	y	y	y	y	y	5 built-in, 8 plug-in Solvers including Gurobi, Xpress-MP, KNITRO, Mosek, SNOPT	y	y	y	y	y	–	All different constraints	y	y	y	y	General non-smooth non-convex	y	y	y	y	y	y	y	–	–	–	SharePoint 2013, Windows Azure	Supports Excel 2013, Excel Web App; solve locally or in cloud; faster on CPUs/GPUs; Constraint Wizard, in-product Live Chat	Global optimization via interval B&B or multistart methods, genetic algorithms, tabu/scatter search, model-based local search
	y	y	y	y	y	Gurobi, Cplex, CBC	–	y	y	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	y	–	–	–
	–	–	–	–	–	QMS includes all popular quantative models in addition to linear programming	y	–	y	y	–	–	–	–	–	–	–	–	–	y	–	y	–	–	y	–	–	–	Software as a service - hosted by QuantMethods	–	Mathematical, network and forecasting models, inventory and production, simulation, decision analysis, dynamic programming, queueing and learning curves.
	y	y	y	y	y	5 built-in, 8 plug-in Solvers including Gurobi, Xpress-MP, KNITRO, Mosek, SNOPT	y	y	y	y	y	–	All different constraints	y	y	y	y	General non-smooth non-convex; chance constraints, et al	y	y	y	y	y	y	y	Robust optimization, stochastic decomposition, sim. optimization	–	–	Windows Azure using Solver Server	Supports Excel 2013; solve on PC or in cloud; faster on CPUs/GPUs; Distribution Wizard, Constraint Wizard, visualization aids	Nonlinear/global optimization, genetic algorithms, tabu/scatter search; Monte Carlo simulation/ risk analysis, decision trees
	y	y	y	y	y	–	–	–	y	–	–	–	–	y		y	y	–	y	y	y	y	y	y	y	Decomposition, network algorithms, concurrent solve	–	y	Public, private, community cloud computing.	Concurrent solve capability; decomposition algorithm; new suite of network optimization and diagnostic algorithms.	Mixed integer, quadratic, and general nonlinear optimization; local search optimization; constraint logic programming; genetic algorithms.
	–	–	–	–	y	GAMS, AMPL, ZIMPL, Comet, G12	y	y	y	y	y	y	–	y	y	y	y	Indicator constraints, logic constraints, cumulative constraints	–	–	–	y	y	y	y	Column generation, branch-and-price, solution counting	y	–	–	Nonconvex MINLP, infeasibility analysis, new presolvers and heuristics, support for scheduling and pseudoBoolean optimization	Framework, customizable via callbacks; reads more than 10 different input formats; HowTos, FAQ, mailing list, examples;
	y	y	y	y	y	13 Solvers including Gurobi, Xpress-MP, KNITRO, Mosek, SNOPT	–	y	y	y	y	y	–	y	y	y	y	General non-smooth non-convex	y	y	y	y	y	y	y	Stochastic programming, robust optimization, simulation optimization	–	y	Windows Azure	Rich tools to build client-server, cloud and SOA applications: Web Services API, JavaScript and PHP support, Solver Server	Monte Carlo simulation and risk analysis
	y	y	y	y	y	PULP, GMPL, COOPR/Pyomo, SimPy, AMPL, GAMS, Gurobi	–	y	y	y	y	y	–	y	y	y	y	–	y	y	y	y	y	y	y	–	y	–	–	Added NEOS support for AMPL & GAMS; added SimPy and COOPR/Pyomo Support; Multiple interface enhancements; bug fixes	–
	–	–	–	–	y	SCIP, COIN-OSI, GAMS	y	y	–	–	–	–	–	–	–	–	–	–	y	y	–	–	y	–	y	Crash basis, scaling, row basis	–	–	–	SoPlex now features an iterative refinement procedure to obtain extended-precision LP solutions.	–

	Software Description															Platforms Supported					Pricing Information							
	Type					Form														Multicore & Multiprocessor Support (List Platforms)		Commercial			Educational			Student/Academic	
	Solver	Modeling Environment	Integrated Solver & Modeling Environ.	Independent Application	C	Procedure Class Library: C++	Procedure Class Library: .NET	Procedure Class Library: Java	Procedure Class Library: Python	Procedure Class Library: MATLAB	Procedure Class Library: R	Procedure Class Library: Other	Source Code	Language?	Add-in To:	PC/Windows	PC/Linux	Mac OS X	Other (specify)	Shared Memory	Distributed Memory	Single Machine	Floating Licenses available	Site Licenses available	Single Machine	Floating Licenses available	Site Licenses available	Unlimited	Size Restricted	
SOPT (Smart Optimizer) 4.3 *SAITECH, inc.*	y	y	y	y	y	y	y	–	–	–	–	–	y	C	Excel	y	y	–	Solaris 9, 10, 11	–	–	Contact	y	y	Contact	y	y	–	–	
Sulum Optimization Tools *Sulum Optimization*	y	y	–	–	y	y	y	–	y	–	–	–	–	–	–	y	y	–	–	–	–	$1,000-$2,500	y	–	Free	y	–	–	–	
SYMPHONY *COIN-OR Foundation*	y	–	–	y	y	y	–	–	–	–	y	–	y	C/C++	–	y	y	y	–	All	All	$0	–	–	$0	–	–	$0	–	
UG *Zuse Institute Berlin*	y	–	–	y	–	y	–	–	–	–	–	–	y	C++	–	–	y	y	–	PC/ LINUX, Mac OS X	PC/ LINUX, Mac OS X	Contact	y	y	Free	y	y	Free	Free	
What'sBest! *LINDO Systems, Inc.*	–	–	y	y	–	–	–	–	–	–	–	VBA	–	–	Excel	y	–	–	–	PC/ Windows	–	Varies / options from $495	–	y	Free educational research licenses available	–	y	Yes	Yes	
XA Professional Linear Programming System *Sunset Software Technology*	y	–	–	y	y	y	y	y	y	y	–	Visual Basis	–	–	Excel	y	y	y	Solaris	Windows, Linux and Solaris	–	Contact	–	–	Contact	–	–	–	–	
YALMIP *Johan L^fberg*	–	y	–	–	–	–	–	–	–	y	–	–	y	MATLAB	–	y	y	y	–	Anything	Anything	Free	–	–	Free	–	–	Free	–	
ZIMPL *Zusee-Institute Berlin*	–	y	–	y	y	–	–	–	–	–	–	–	y	C	–	y	y	y	AIX, Solaris	–	–	$0	–	–	$0	–	–	$0	–	

	Data Compatibility					Solvers or Modeling Environments			Formulations Supported										Algorithms/Methods							Cloud /Remote Access Availability			New Features (Since June 2011)	Other Techniques (Available as part of the package)
									Variable Types					Constraint & Objective Types																	
	Reads Spreadsheet Files	Writes Spreadsheet Files	Reads Database Files	Writes Database Files	Reads and Writes Text Files	Solvers or Modeling Environments that Link to this Product	Bundeled as a Single Package	Available Separately	Integer, Binary	Semi-continuous, Semi-integer	Arbitrary Discrete (SOS1)	Piecewise Linear (SOS2)	Other	Convex Quadratic	Conic Quadratic	General Convex	General Nonlinear	Other	Primal Simplex	Dual Simplex	Interior-Point	Branch-and-Cut	Presolve	Heuristic Search	Infeasibility Analysis	Other	NEOS	Amazon	Other		
	y	y	–	–	y	AMPL	–	–	y	–	–	–	–	y	–	y	y	–	y	–	y	–	y	y	y	–	–	–	–	–	Extended rounding and search capabilities are used find feasible solutions quickly to large-scale integer programs. Automatical cuts can be generated with search scope control by user parameters.
	–	–	–	–	y	GAMS, MPL	–	–	y	–	–	–	–	–	–	–	–	–	–	y	–	y	y	y	–	–	–	y	Private Clouds available too	New mixed integer optimizer and a integrated cloud-server solution.	–
	–	–	–	–	y	AMPL, GAMS	–	y	y	–	–	–	–	–	–	–	–	–	y	y	–	y	y	y	–	–	y	–	–	–	Biobjective integer programming, Sensitivity analysis, warm starting
	–	–	–	–	y	SCIP	y	y	y	y	y	y	–	y	y	y	y	–	–	–	–	y	y	y	–	–	–	–	–	–	–
	–	–	–	–	y	LINDO API	y	–	y	y	y	y	–	y	y	y	y	Can explicitly identify convex constraints, Geometric Programs	y	y	y	y	y	y	y	Stochastic & Chance Constraints, Guaranteed Global, Nonlinear Least Squares	–	–	–	Support for 200+ mathematical and probability functions, histogram generation, branch-and-price solver, multicore support.	Genetic algorithm, decomposition algorithm, Sprint/sift technique, auto detect problem type.
	y	y	y	y	y	AIMMS, GAMS, AMPL and MPL	–	y	y	y	y	–	–	y	–	–	–	–	y	y	y	–	y	y	y	–	–	–	–	1) Improved MIP solving algorithm. 2) A dual solution to Clients who wish to use the Gurobi Solvers without any application modification.	You can very easily to use XA Java API in a Client and/or Server environment. Now you can solve LP/MIP models directly from your web applications via our .NET interface, or from your web HTML code.
	y	y	y	y	y	bintprog, bonmin, bpmpd, cbc, cdd, clp, cplex, csdp, dsdp, filtersd, fmincon, glpk, gpposy, et al	–	–	y	y	y	y	Uncertain	y	y	y	y	socp, semidefinite, posynomial	–	–	–	–	–	–	–	–	–	–	–	Bilevel programming, Symbolic KKT systems	Convex programming, Robust programming, Determinant optimization, Sum-of-squares, Global optimization, Geometric programming
	–	–	–	–	y	SCIP, lp_solve	y	y	y	y	y	y	–	y	–	y	y	–	–	–	–	–	–	–	–	–	–	–	–	Support for MINLP	–

Software Survey, *continued from 41*

Essentially all LP software packages now take full advantage of available memory (using 64-bit processors), as well as available disk space where appropriate.

Pricing and Distribution

The table shows single commercial licenses running from hundreds to thousands of dollars, but as most vendors have a considerable range of license types and pricing arrangements, it is advisable to get a quote or consult a complete price sheet before arriving at any conclusions about costs. The most popular MIP solvers do tend to be at the high end of the price range. Special terms are often available for multiple purchases and for **site licenses** or **floating licenses** (which permit a certain number of copies to be used anywhere in a user's network). Since solver performance varies considerably from problem to problem and from product to product, buyers are well advised to benchmark problems of interest before deciding which products are likely to offer the best value.

For **educational** purposes and **academic** research, an increasing number of full-featured commercial systems are available free under reasonable conditions. Free size-limited **student** (or demo) versions are also available for experimentation with small problem instances; several modeling systems offer conveniently packaged demo versions with one or more solvers.

Where free versions are not available, both modeling language and solver developers will arrange to provide full versions of their software for testing for a limited time. Of course, open-source packages are free for any use.

Convenient **Remote Access** or **Cloud** services are increasingly an alternative to local installations of optimization software. Many modeling systems and solvers have long been available for testing by submission through the Internet to the **NEOS Server** (www.neos-server.org). As a free service, NEOS does not guarantee confidentiality or performance, but it is appropriate for many projects and has been averaging more than 35,000 requests a month. Commercial cloud availability has expanded considerably since the last survey, using **Amazon EC2** and other cloud platforms. Services are quite varied at this stage, ranging from creation of virtual machines with preinstalled optimization software to acceptance of NEOS-style submissions of individual optimization jobs.

Problem Types

Many packages seek to address their users' needs by supporting varied specializations and generalizations of LPs and MIPs.

In the area of discrete optimization, the ideas underlying branch-and-bound search for integer programming are sufficiently powerful to handle broader classes of constraint types.

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Indeed, MIP solvers have long accommodated variables that take values from an arbitrary list (via special ordered sets of type 1 or **SOS1** search rules) and objectives or constraints that incorporate non-convex piecewise-linear terms (via **SOS2** rules). Many solvers also have special search rules to help with **semi-continuous** or **semi-integer** variables, which must lie either at zero or in a designated positive range. Additional kinds of logical constraints, such as if-then and all-different, are becoming more common as specialized search techniques are adapted from the related but distinct realm of constraint programming software. The distance between integer and constraint programming is thus slowly narrowing, though it will not likely disappear anytime soon.

Convex quadratic objectives and constraints, in continuous or integer variables, are another popular extension; they can be solved by extensions to LP interior-point and branch-and-bound methods, though not always as easily. At the least, this category includes the classic positive semidefinite quadratic formulations that describe elliptical regions. An increasing number of solvers also accept **conic quadratic** formulations that specify various cone-shaped convex regions. These latter are particularly versatile because a variety of non-quadratic and even non-convex function classes admit translations to them, thus extending the range of applications that can be addressed.

Although this survey is focused on LP, quite a few of the listed products can also handle **general convex, general nonlinear** and **other constraint types** including complementarity, semi-definite, logical and chance constraints. The listing should not be considered exhaustive with respect to these types, however, as other kinds of high-quality solvers can be applied to them as well.

Algorithms

Solution methods have continued to be refined for speed and reliability. For linear programs a choice between **primal simplex, dual simplex** and **interior-point** methods is standard. The table records MIP solvers that provide **branch-and-cut, pre-solve** and **heuristic search** features, all of which have become quite standard. Also many of those listed provide some form of **infeasibility analysis** as an aid to model development.

Much of the improvement to MIP solvers has come through an accumulation of ideas for reducing problem size, tightening bounds and finding better solutions. Inevitably this has led to very long option lists; close attention to solver documentation can suggest which option settings might improve performance on a particular problem, but often only a fraction of the possibilities can be tested. Thus an important aspect of any MIP solver is its choice of default settings that adapt to characteristics of the problem at hand; a few solvers also provide automated "tuning" features that can suggest to the user which options to set. For difficult problems some careful benchmarking is ultimately necessary to make a reasoned choice among solvers and options.

Prominent in the list of **other algorithms** are a variety of extensions that fall under the general heading of stochastic optimization. These include classic recourse problems given distributions or scenario trees for the data, explicit chance constraints and other formulations that seek a solution that is robust in the face of uncertainty. Implementations vary considerably due to the great variety of problem types.

What's New

Among the reported **new features** since the previous survey, purely algorithmic enhancements are a minority. Many new features involve more convenient or extensive interfacing to other systems and environments. Also common are new facilities that allow algorithms to take advantage of more powerful computer and network architectures, notably multiprocessor and cloud environments as mentioned earlier. Finally there is evidence of a broadening of systems to recognize and efficiently handle additional problem types and structures, thereby permitting an individual modeling system or solver to address a broader variety of applications. ORMS

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