Technische Universität Berlin
Institut für Mathematik

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Mathematics of Infrastructure Planning

PD Dr. Ralf Borndörfer
Prof. Dr. Martin Grötschel

Exercise sheet 13

Deadline: Tutorial session only

Exercise 47. **(Tutorial session)**

Peter wants to buy a new computer but has little money. To save costs, he decides to purchase individual parts that he plans to assemble himself. In a tedious internet search, he identifies five suppliers that sell the parts that he wants at the following prices (in Euros):

supplier	1	2	3	4	5
mainboard	100	120	150	120	110
CPU	190	200	205	190	210
RAM	65	50	45	50	65
harddisk	105	80	105	100	105
frame	50	45	50	40	40

All suppliers charge (in Peter's opinion) ridiculously high delivery costs independent of the size of the order, namely, suppliers 1, 2, and 3 charge 20 Euros, supplier 4 charges 15 Euros, and supplier 5 charges 10 Euros. Clenching his teeth, Peter recalls his math class. After all, a computer is also a kind of infrastructure. At least, he can … yeah, what can he do?

a) Develop an IP model for Peter's computer acquisition problem.
b) What kind of model is this?
c) Solve the model. What is the optimal solution?

Exercise 48. **(Tutorial session)**

In the (Metric) Capacitated Facility Location problem, we are given a number u_i for each facility i, and a facility i can serve at most u_i clients. Adjust the IP formulation for the (Metric) Uncapacitated Facility Location Problem to this situation and show that the integrality gap between the LP and the IP optimium is unbounded.

Exercise 49. **(Tutorial session)**

Consider the following metric uncapacitated facility location problem. There are k facilities, all with opening costs p^k and all located at the same place, and $k-1$ groups of customers $S_1, \dots, S_{k-1}$. Group S_i consists of p^{k-i+1} customers, all located at equal distances from the facilities, the sum of the distances being $\sum_{j=1}^{i} p^{j-1}$.

a) The greedy algorithm chooses all facilities, incurring a cost of $\Omega(kp^k)$.
b) The optimal solution chooses exactly one facility, incurting a cost of $p^k + O(p^{k-2})$.
c) The ratio of the costs is $\Omega(k)$.
d) The number n of cities is $n = \Omega(p^k)$.
e) For $p = \log n$, the cost ratio is $\Omega(\log_p n) = \Omega\left(\frac{\log n}{\log p}\right) = \Omega\left(\frac{\log n}{\log \log n}\right)$.
f) The greedy algorithm for the metric uncapacitated facility location problem has a performance guarantee which is at best $\Omega\left(\frac{\log n}{\log \log n}\right)$.