Technische Universität Berlin
Institut für Mathematik

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Mathematics of Infrastructure Planning

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Exercise sheet 4

Deadline: Thu, May 10, 2012, in the tutorial session

Mobile Phone Network Coverage

Given a set of locations $L = \{r_1, ..., r_s\}$ representing the region to be covered by a system of antennas of a mobile telephone network. Given, moreover, a set $A = \{a_1, ..., a_n\}$ of potential antenna locations and, for each antenna location $a \in A$, a set $T_a = \{t_{1a}, ..., t_{qa}\}$ of feasible "antenna technologies". For each antenna $a \in A$, each associated antenna technology $t \in T_a$, and each location $r \in L$, a non-negative rational number v_{atr} is given, indicating how well location $r \in L$ is covered by antenna $a \in A$ when equipped with technology $t \in T_a$. If $v_{atr} = 0$ then r is not reached from antenna a by technology t.

Exercise 9. **5 points**

Formulate an integer program that chooses at most one technology for each antenna and maximizes the coverage of the locations representing the region.

Exercise 10. **5 points**

Now assume that the available technologies are the same for all antenna locations, briefly $T = \{1, ..., q\}$, but that for each antenna $a \in A$, a set $F_a \subseteq T$ of technologies is given that are "forbidden" for this antenna. Moreover, for any pair $a, b \in A$ of antenna locations, an interference value c_{ab} is given that indicates the "interference" occurring in the overlap of the "antenna cells" of a and b when in both locations a and b the same antenna technology is installed. The overlap region of two antenna cells is the set of locations $r \in L$ where both antennas simultaneously have positive coverage.

Formulate an integer program that guarantees coverage of every location in L and that minimize total interference.

Exercise 11. **5 points**

Suppose now that, in addition to the data of Exercise 10, values w_{at} have to be taken into account that indicate the costs occuring when technology $t \in T$ is installed at antenna location $a \in A$. Design an optimization model that produces an assignment of technologies to antenna locations such that the total costs are small as well as the total interference.

Exercise 12. **5 points**

Suppose now that, in addition to the data of Exercise 10, values d_{ab} are given that indicate the "neighbor interference" which occurs when, for the technology $s \in T$ installed at $a \in A$ and the technology $t \in T$ installed at $b \in A$, the following holds: $|s - t| \leq 1$. Incorporate this requirement into your optimization model.

Exercise 13. **10 points**

Provide your opinion about the four models explained above. Which looks reasonable, practical, implementable? Which would you suggest to a mobile phone company? Are there additional constraints that should be addressed?