Technische Universität Berlin
Institut für Mathematik

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# Mathematics of Infrastructure Planning

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## Exercise sheet 11

Deadline: Thu, Jun 28, 2012, **23:59**, `mailto:borndoerfer@zib.de`

**Exercise 40.** **10 points**

Consider the uncapacitated facility location problem

$$\begin{array}{llrcll}
\text{(UFL)} & \min & \displaystyle\sum_{i\in I} f_i y_i + \sum_{ij\in A} d_{ij}x_{ij} & & & \\
\text{(i)} & & \displaystyle\sum_{i\in I} x_{ij} & \geq & 1 & \forall j\in J \\
\text{(ii)} & & y_i & \geq & x_{ij} & \forall ij\in A \\
\text{(iii)} & & y_i & \in & \mathbb{Z}_+ & \forall i\in I \\
\text{(iv)} & & x_{ij} & \in & \mathbb{Z}_+ & \forall ij\in A,
\end{array}$$

where $I=\{1,\ldots,m\}$, $J=\{1,\ldots,n\}$, $A\subseteq I\times J$, $f\in\mathbb{R}^I_+$, and $d\in\mathbb{R}^A_+$.

Prove:

a) (UFL) has an optimal 0/1-solution.
b) (UFL) $\iff$ (UFL) (i), (ii), (iii), $x_{ij}\geq 0\ \forall ij\in A$.

**Exercise 41.** **10 points**

Consider the following set covering problem associated with model (UFL) of exercise 40:

$$\begin{array}{llrcll}
\text{(SCP)} & \min & \displaystyle\sum_{(i,J')\in\mathcal{J}} c_{(i,J')}z_{(i,J')} & & & \\
\text{(i)} & & \displaystyle\sum_{J'\ni j} z_{(i,J')} & \geq & 1 & \forall j\in J \\
\text{(ii)} & & z_{(i,J')} & \in & \{0,1\} & \forall (i,J')\in\mathcal{J}.
\end{array}$$

Here, $J(i):=\{j\in J : ij\in A\ \forall i\in I\}$, $\mathcal{J}:=\{(i,J') : i\in I, \emptyset\subsetneq J'\subseteq J(i)\}$, and $c(i,J'):=f_i+\displaystyle\sum_{j\in J'} d_{ij}\ \forall (i,J')\in\mathcal{J}$.

Prove:

a) There is a one-to-one correspondence between optimal 0/1-solutions of (UFL) and (SCP).
a) UFL is APX-hard. **Hint:** SCP is APX-hard.

**Exercise 42.** **10 points**

Modify the LP rounding Algorithm 3.2.3 for the MUFL as follows:

i) Change the definition of $x'$ in the filtering step to

$$x'_{ij} \leftarrow \begin{cases} 0, & \text{if } i \notin N_j(\beta) \\ \alpha \bar{x}_{ij}, & \text{else} \end{cases}$$

for an approriate $\alpha$ of your choice.

i) Replace in the LP rounding step $N_j$ by $N_j(\beta) := \{i \in I : x_{ij} > 0 \wedge d_{ij} \leq \beta D_j\}$.

i) What is the resulting approximation ratio depending on $\beta$?

i) What is the best approximation ratio that can be obtained by modifying $\beta$?