Technische Universität Berlin
Institut für Mathematik

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Mathematics of Infrastructure Planning

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Exercise sheet 10

Deadline: Thu, Jun 21, 2012, **23:59**, `mailto:borndoerfer@zib.de`

Exercise 37. **10 points**

Consider the vehicle scheduling problem in the file `arcs.dat`:

```
#  dh  tail  head  depot    cost
a   0     0     1      2    1800
a   1     0     2      2    1800
a   2     0     3      0    1320
a   3     0     4      2    1800
...
```

File `arcs.dat`.

The file has the following format. A line starting with `#` contains comments. A line starting with `a` defines an arc in terms of five numbers, namely, a deadhead trip index `dh`, a start node `tail`, an end node `head`, a depot or vehicle type `depot`, for which this arc is feasible, and a `cost`. There can be several arcs with the same deadhead index, tail, and head, but different depots; these arcs model the feasibility of the same deadhead trip for serveral types of vehicles. `dh`, `tail`, `head`, and `type` are integer numbers starting with 0, costs are nonnegative integer numbers. The artificial node 0 denotes the beginning and the end of the vehicle rotations, the other nodes represent timetabled trips. The problem in file `arcs.dat` involves 11 depots (= vehicle types), 7 130 timetabled trips, 7 131 nodes, 75 137 deadhead trips, and 126 992 arcs.

a) Fomulate and solve the vehicle scheduling problem using Zimpl and `Scip`.
b) What is the optimal objective value of the IP and the LP relaxation?
c) How many vehicles are used in each case?
d) Is this the minimum fleet size (= number of vehicles)?

Exercise 38. **10 points**

This exercise continues Exercise 37. Files `tripduals.dat` and `flowduals.dat` contain multipliers for the flow and the flow conservation constraints of a multi-commodity flow formulation of the vehicle scheduling problem:

```
# node    dual
v    1    1740.000000
v    2    2280.000000
v    3    1200.000000
v    4    2040.000000
...
```

File `tripduals.dat`.

```
# node depot     dual
v    0     0      420.000000
v    0     1        0.000000
v    0     2      240.000000
v    0     3     3330.000000
...
```

File `flowduals.dat`.

Each noncomment line of file `tripduals.dat`, starting with `v`, contains a node index (= index of a timetabled trip, different from 0) and an associated multiplier. Each noncomment line of file `flowduals.dat` contains a node index, a depot (= vehicle type), and an associated multiplier. The multipliers can be arbitrary floating point numbers.

a) Construct an m-depot relaxation of the vehicle scheduling problem by ignoring the flow constraints. Show that this m-depot relaxation is a minimum cost flow problem. What is the optimal objective value?
b) Improve the m-depot relaxation by adding a Lagrange relaxation of the flow constraints, using the multipliers from file `tripduals.dat`. Show that this Lagrangean relaxation with fixed multipliers is a min-cost flow problem. What is the optimal objective value?
c) Construct a 1-depot relaxation of the vehicle scheduling problem by pretending that all arcs are feasible for vehicle type 1. Show that the 1-depot relaxation is a min-cost flow problem. What is the optimal objective value?
d) Improve the 1-depot relaxation by adding a Lagrange relaxation for the flow conservation constraints associated with the individual vehicle types, using the mulotipliers in file `flowduals.dat`. Show that this Lagrangean relaxation with fixed multipliers is also a min-cost flow problem. What is the optimal objective value?
e) Which relaxation is better?

Exercise 39. **10 points**

Use `Zimpl` to implement a set covering model for a vehicle and crew scheduling problem. There is data for three days: Saturday (`dsp1-sa`), Friday (`dsp1-fr`), and Thursday (`dsp1-th`). The objective coefficients and the indices of the nonzero elements of the columns are listed in the data files in the format `c obj col` and `r row col`, where row and column are indices, and `obj` is a floating point number.

a) Use `Zimpl` to implement a set covering model. **Hint:** Use the template file `dsp1-skeleton.zpl`; it is used with a command line such as this:

```
zimpl -DFILE="<FILENAME>" dsp1-skeleton.zpl
```

   The define specifies a file to read.
b) Solve these models using `SCIP`.
c) Compare the solutions of LP-relaxations to those of the IPs.
d) Solve the set partitioning variant of the models.