Technische Universität Berlin
Institut für Mathematik

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Mathematics of Infrastructure Planning

PD Dr. Ralf Borndörfer
Prof. Dr. Martin Grötschel

Exercise sheet 5

Deadline: Thursday, May 24, 2012 (please submit your solution by email to: `borndoerfer@zib.de`)

Connectivity and node degrees

A lot of network design problems coming up in practice require certain connectivity properties or have bounds on node degrees. It is therefore important to know basic models with which such constraints can be formulated. A glimpse of requirements of this type are the subject of this exercise sheet.

We are given a graph $G = (V, E)$ with rational edge weight $c_e, e \in E$. We want to find:

(a) a subset $M \subseteq E$ of maximum weight such that every node of V is contained in at most 2 edges,

(b) a subset $C \subseteq E$ of minimum weight such that every node of V is contained in at least 2 edges,

(c) a subset $C \subseteq E$ of minimum weight such that (V, C) is 2-edge connected,

(d) a forest of maximum weight.

Exercise 14. **10 points**

Formulate (a), (b) and (c) as integer programs using 0/1-variables $x_e, e \in E$, and no other auxiliary variables.

Exercise 15. **10 points**

Search the literature to find out whether (a) and/or (b) and/or (c) are solvable in polynomial time or NP-hard.

Exercise 16. **10 points**

Determine (by literature search) for at least one of the problems (a), (b), (c) a linear program such that the vertices of the polyhedron determined by the constraints of the linear program are in one-to-one correspondence with the 0/1-solutions of the integer program that you formulated in exercise (14).

Exercise 17. **10 points**

Find (by literature search) a system of inequalities that provides a complete linear description of
$F(G) := conv\{\chi^T | (V, T) \text{ is a forest}\}$.