Outline

This introductory textbook is, in the first place, addressed to students of mathematics, computer science, science, and engineering. In the second place, it is also addressed to computational scientists already on the job who wish to get acquainted with modern concepts of Numerical Analysis and Scientific Computing on an elementary level via personal studies.

The book is divided into nine chapters, including associated exercises, a software list, a reference list, and an index. The contents of the first five and of the last four chapters are each closely related.

In **Chapter 1** we begin with *Gaussian elimination* for linear systems of equations as the classical prototype of an algorithm. Beyond the elementary elimination technique we discuss pivoting strategies and iterative refinement as additional issues. **Chapter 2** contains the indispensable *error analysis* based on the fundamental ideas of J. H. Wilkinson. The condition of a problem and the stability of an algorithm are presented in a unified framework, well separated and illustrated by simple examples. The quite unpopular "ϵ-battle" in linearized error analysis is avoided—which leads to a drastic simplification of the presentation and to an improved understanding. A stability indicator arises naturally, which allows a compact classification of numerical stability. On this basis we derive an algorithmic criterion to determine whether a given approximate solution of a linear system of equations is acceptable or not. In **Chapter 3** we treat orthogonalization methods in the context of Gaussian *linear least-squares problems* and introduce the extremely useful calculus of pseudo-inverses. It is immediately applied in the following **Chapter 4**, where we present iterative

methods for systems of nonlinear equations *(Newton method)*, nonlinear least-squares problems *(Gauss-Newton method)*, and parameter-dependent problems *(continuation methods)* in close mutual connection. Special attention is paid to modern affine invariant convergence theory and iterative algorithms. **Chapter 5** starts with a condition analysis of *linear eigenvalue problems* for general matrices. From this analysis, interest is naturally drawn to the real symmetric case, for which we present the power method (direct and inverse) and the QR-algorithm in some detail. Into the same context fits the singular value decomposition for general matrices, which is of utmost importance in application problems. As an add-on in this second edition, we finally consider stochastic eigenvalue problems, which in recent years have played an increasing role, especially in cluster analysis.

The second closely related chapter sequence begins in **Chapter 6** with an extensive theoretical treatment of *three-term recurrences*, which play a key role in the realization of orthogonal projections in function spaces. The condition of three-term recurrences is represented in terms of discrete Green's functions—thus paving the way toward mathematical structures in initial and boundary value problems for differential equations. The significant recent spread of symbolic computing has renewed interest in *special functions* also within Numerical Analysis. Numerical algorithms for their fast summation via the corresponding three-term recurrences are exemplified for spherical harmonics and for Bessel functions. In **Chapter 7** we start with classical polynomial *interpolation* and *approximation* in the one-dimensional case. We then continue over Bézier techniques and splines up to methods that nowadays are of central importance in CAD (**C**omputer-**A**ided **D**esign) or CAGD (**C**omputer-**A**ided **G**eometric **D**esign), disciplines of computer graphics. Our presentation in **Chapter 8** on *iterative* methods for the solution of *large* symmetric systems of linear equations benefits conveniently from Chapter 6 (three-term recurrences) and Chapter 7 (minimax property of Chebyshev polynomials). The same is true for our treatment of the Lanczos algorithm for large symmetric eigenvalue problems.

Finally, **Chapter 9** has deliberately gotten somewhat longer: it bears the main burden of presenting principles of the numerical solution of ordinary and partial differential equations without any technicalities at the simplest possible problem type, which here is numerical quadrature. We start with the historical Newton-Cotes and Gauss-Christoffel quadrature. As a first adaptive algorithm, we introduce the classical Romberg quadrature, wherein, however, only the approximation order can be varied. The formulation of the quadrature problem as an *initial value problem* offers the opportunity to work out an *adaptive Romberg* algorithm with variable order *and* step-size control; this approach opens the possibility to discuss the principle of *extrapolation methods*, which play a key role in the numerical solution of ordinary differential equations. The alternative formulation of the quadrature problem as a *boundary value problem* is used for the deriva-

tion of an *adaptive multigrid* quadrature; in this way we can deal with the adaptivity principle behind multigrid methods for partial differential equations in isolated form—clearly separated from the principle of fast solution, which is often predominant in the context of partial differential equations.