Exercise 13: Solving VRP with Concorde

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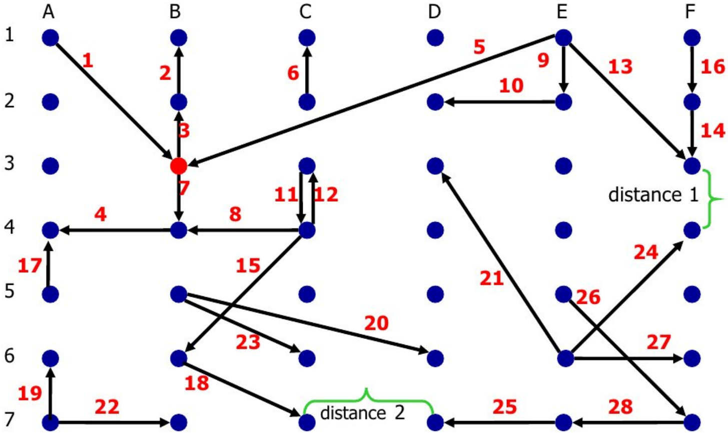
Concorde

http://www.tsp.gatech.edu/concorde.html

Vehicle Routing Problem

- Instance from exercise 5



Vehicle Routing Problem

- Tour length is unlimited
- VRP with pick-up and delivering

Given

- Directed graph $G(V, A)$, depot $0 \in V$
- Distance function $d : A \to \mathbb{R}$
- D set of requests, each $r \in D$ with origin $o_r \in V$ and $d_r \in V$ destination
- m number of vehicles, capacity one, additional fix cost C per used vehicle

Problem

Find vehicle routes that

- originating and terminating at a depot,
- does not exceed the vehicle's capacity,
- every request is served by exactly one vehicle and
- minimize the vehicle fleet and the sum of distances

m-ATSP

Given

- Directed graph $G(V, A)$, home city $0 \in V$
- m-salesman with fixed cost C per deployed salesman
- Distance function $d : A \to \mathbb{R}$

r-tour

A set of elementary cyles $Z_k, k = 1, \ldots, r$ is called an r-tour if

- every cycle contains node 0 and
- every node $v \in V/\{0\}$ belongs to exactly one of the r cycles.

Cost of a cycle $Z_k = (u_1, \ldots, u_l)$

$$d(Z_k) = \sum_{i=1}^{l-1} d(u_i, u_{i+1}) + d(u_l, u_1)$$

m-ATSP

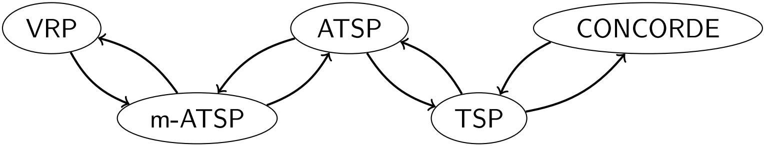
Problem

Find a r-tour, $r \in \{1, \ldots, m\}$ that minimize

$$\sum_{k=1}^{r} d(Z_k) + r \cdot C$$

Roadmap

- Concorde could only solve the symmetric TSPs
- VRP hat to be transformed into an TSP



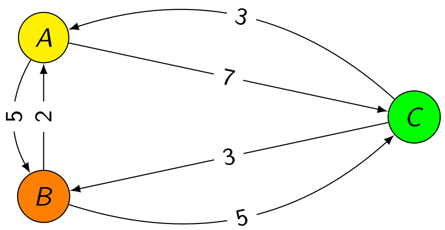
m-ATSP to 1-ATSP (Bellmore, Hong 1974)

$G(V, A)$ with m salesman, depot at node 0, fixed cost C

expanded graph G(V', A')

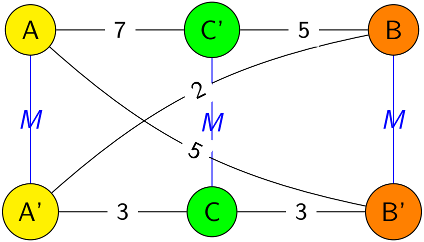
- Nodes V':
  - V
  - $\bar{V} := \{1', \ldots, (m-1)'\}$
- Arcs A':
  - A
  - $\{(u, \bar{v}) \in V \times \bar{V} \mid (u, 0) \in A\}$
  - $\{(\bar{u}, v) \in \bar{V} \times V \mid (0, v) \in A\}$
  - $\{(u', (u-1)') \mid u \in \{1, \ldots, m-1\}\} \cup \{1', 0\}$
- Distance function d'
  - $d'(u, v) = d(u, v) \quad \forall u, v \in V/\{0\}$
  - $d'(\bar{u}, v) = d(0, v) + \frac{1}{2}C \quad \forall \bar{u} \in \bar{V}, v \in V/\{0\}$
  - $d'(u, \bar{v}) = d(u, 0) + \frac{1}{2}C \quad \forall u \in V/\{0\}, \bar{v} \in \bar{V}$
  - $d'(u', (u-1)') = 0 \quad \forall u \in \{2, \ldots, (m-1)\}$
  - $d'(1', 0) = \frac{1}{2}C$

- Example: 3 nodes



Asymmetric to Symmetric

- $V = \{v_1, v_2, \ldots, v_n\}$ set of nodes of asymmetric TSP
- Extend graph by a copy for each node $V' = \{v'_1, v'_2, \ldots, v'_n\}$
- It is:
  - $d'(v, v') = M \ll min_{(u,v)\in V\times V} d(u, v)$,
    $d'(u, v') = d(u, v)$,
    $d'(u', v) = d(v, u)$



Remarks

- Data instance
  - 4-tupel of coordinates x_o y_o x_d y_d of origin o and destination d
  - first 4-tupel is the depot
  - distance of two points (x_1, y_1), (x_2, y_2) defined by

$$2|x_2 - x_1| + |y_2 - y_1|$$

- VRP to m-ATSP
  - the fixed cost per vehicle could be modeled by adding C to the cost of each outgoing arc from the depot