

Nonlinear Optimization

<http://www.zib.de/weiser/NichtlineareOptimierung/>
Homework 6

Due: Friday, June 5, 2020

Assignment 1 (4 points):

Consider $f(x, y) = \sin(x) \sin(y)$. To which points can the ordinary Newton method and the ordinary steepest descent method converge? Sketch the basins of attraction for different limit points.

Assignment 2 (4 points):

For the Rosenbrock function

$$f(x) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2,$$

draw the level sets of the quadratic model

$$m(\delta x) := f(x) + f'(x)\delta x + \frac{1}{2}\delta x^T f''(x)\delta x$$

in $x = (0, -1)$ and $x = (0, 0.5)$. Using this, sketch and justify the trust region path for $\rho \in [0, 2]$.

Assignment 3 (4 points):

In the lecture videos three approaches to handle nonconvex problems were presented (hessian modification, trust region and cubic upper bound). A fourth possibility would be to minimize the quadratic problem

$$f(x) + f'(x)\delta x + \frac{1}{2}\delta x^T f''(x)\delta x + \kappa \|\delta x\|_M^4.$$

Show that whenever $f''(x) + \eta M$ is s.p.d. for $\eta \geq 0$, and $\delta x(\eta)$ the Levenberg-Marquardt step, there is a $\kappa \geq 0$ such that the quadratic step $\delta x(\kappa) = \delta x(\eta)$.